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RELATIONSHIP OF ARITHMETIC PROBLEM SOLVING AND REFLECTIVE-IMPULSIVE COGNITIVE STYLES IN THIRD-GRADE STUDENTS[1]

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Summary.—Different individuals approach mathematical problems in a variety of ways, with these different approaches also reflected in over-all cognitive styles. This investigation had two purposes, first, to determine whether good and poor arithmetic problem solvers differ substantially in cognitive style, and second, to determine whether the students, after training in techniques of solving arithmetic problems, improve their performance with no significant change in cognitive style. A total of 98 third graders participated (mean age 8.1 yr.; 50 boys, 48 girls). The Matching Familiar Figure Test was used to classify the students by cognitive style as either Reflective or Impulsive. Students also were given training with different problem-solving exercises for different arithmetic problems. The training program in problem-solving strategies did not improve performance on arithmetic problems for Reflective students; however, Impulsive students' performance did improve after training.

Individuals approach mathematical problems in a variety of ways, and these approaches are also reflected in over-all cognitive style. The concept of cognitive styles presupposes individual variation in the style of perceiving, recalling, and thinking, in other words, different ways of processing information. The problem-solving literature discusses different (but not mutually exclusive) types of cognitive styles (Witkin & Goodenough, 1991). In particular, Reflective or Impulsive styles are considered to be two cognitive styles originating from the studies of Kagan, Moss, and Sigel (1963) on conceptual strategies.

Mathematical problem solving requires complex processing as well as a detailed elaboration of the sequence of steps which lead to a correct solution, so Mathematical problem solving possesses the characteristics of a test of analytical and information-processing styles of children. One might expect that the subjects' cognitive style should bear a special relation to the problem's results.

Previous research has shown a certain similarity between academic

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achievement and certain cognitive styles (Kornbluth & Sabban, 1981; Lederma, Gess-Newsome, & Zeidler, 1992; Abouserie, 1994; Martinsen, 1994). We have designed a study in which the dependent variables are not referred to the students' grades but to the scores obtained by the children within a pool of arithmetic problems.

This investigation had two purposes: first, to assess whether good and poor arithmetic problem solving is related to cognitive style classified as Reflective or Impulsive, and second, to judge whether the students after training in techniques of arithmetic problem solving who improve their performance show significant changes in the cognitive style. This comparison allows assessment of whether mathematical problem solving is related to cognitive style. Finally, it will be possible to assess whether training in mathematical problem solving affects the students' scores on the Matching Familiar Figures Test.

Method

Subjects

A total of 98 third graders, with an average age of 8.1 yr., (range, 8.5 to 9.4 yr.) from the public school district within the city of Cádiz, Spain, participated. The students were divided into two groups, a control group of 25 boys and 24 girls and an experimental group of 25 boys and 24 girls. Each group had an equivalent number of subjects classified as having either Impulsive or Reflective style. The students of a particular classification were randomly assigned to their respective groups. All students came from a lower-middle socioeconomic background, had IQs between 85 and 104 on the Raven Scale of Intelligence, achievement scores in accord with grade and age in most academic areas, and were nominated by their classroom teachers as not having attentional, behavioral, or productivity problems. No significant differences on mathematical problem solving between subjects were found on a pretest that involved 25 third grade mathematical problems from the standard Spanish curriculum.

Procedure

To classify the students' cognitive style, the Matching Familiar Figures Test was used (Cairns & Cammock, 1978). This test presents a total of 20 items in which a stimulus, e.g., a scissors or telephone equipment, serves as a model. Students must compare the model with six other stimuli, e.g., six scissors or six telephones, which are similar but not identical to one another. Only one of the pictures is exactly the same as the sample. Students must indicate which one of the six presented stimuli is exactly the same as the sample. Latencies and number of errors that students made on each item were recorded.

Test procedure.—During the test, students were seated in a comfortable classroom environment and, after a period of adjustment, were informed that to assess their arithmetic skills, they would be given a test. At this time, the Matching Familiar Figures Test was administered. The administration allowed the children as much time as they needed. Some children took as little as 12 min. to solve the 20 items, and those who needed longest took 25 min. The test was conducted in the presence of one of the researchers. The cut-off scores for Reflective and Impulsive cognitive styles on the Matching Familiar Figures Test, and means scores for Experimental and Control groups are presented in Table 1.

TABLE 1

MEANS, STANDARD DEVIATIONS, LATENCIES, AND ERRORS ON MATCHING FAMILIAR FIGURES TEST AND CUT-OFF SCORES FOR REFLECTIVE AND IMPULSIVE COGNITIVE STYLES FOR EXPERIMENTAL AND CONTROL GROUPS

Cognitive Style	Experimental Group				Control Group			
	Latency, sec.		Errors		Latency, sec.		Errors	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
	18.43	6.50	13.78	6.96	14.44	7.17	15.36	7.69
Reflective	>18.43		<13.78		>17.44		<15.36	
Impulsive	<18.43		>13.78		<17.44		>15.36	

Arithmetic testing.—The arithmetic word problems involved were a series of 62 third-grade standard problems using six problems of each of the following formats: Change, Compare, and Equalize; 2 of Combine; 3 of Part–Whole Structure, Large Escalates, Small Escalates; and 2 of Cartesian Product (see Appendix, p. 186). The problems used were of two different configurations. In one, the problems were formulated with small integers so that the products always had a result less than 10 and to which the students were able to respond freely. In the other configuration, problems were presented with large numbers (>10) and with the correct operation one of the four basic alternatives. These two configurations were designed to differentiate whether a wrong solution was based on a wrong operation or only on a simple computation error. There were 62 problems; 31 with large numbers, and 31 with small numbers.

The arithmetic problems were administered in five different 1-hr. sessions. Problems were presented in a random order within the earlier-mentioned categories. In each session, the researcher provided each subject a response sheet on which were printed the selected problems. The sheet contained problems using four basic mathematical operations, either problems with large numbers or problems with small numbers. There was no time limit imposed for solving the problems within the 1-hr. session.

The Experimental group was given a training program which included different problem-solving exercises for Additive problems (Change, Compare, Equalize, Combine, Part–Whole Structure), and Multiplicative problems (Large Escalates, Small Escalates, Isomorphism, and Cartesian Product). Each problem was presented in three different ways. Manipulative Phase was divided into three periods: first, thinking about the problem with no numbers, representing the problem with tokens, and solving the problem, second: pictures and sketching, and third, symbolic representation. The training strategies used in the program were re-reading the problem, underlining the problem's question, and rebuilding problems with large numbers into problems with smaller numbers.

The dependent variables for this experiment consisted of the number of correctly solved problems. The training program for students in the Experimental Group involved five 1-hr. general instruction sessions on heuristic problem solving, according to Polya's model (1957), which was adapted by the researchers for use with third-grade students. The Experimental Group also received 22 instructional sessions in problem solving using examples from the two types of problems. The length of each session was 13 min., with a total of 25 hours spent in instruction. These training sessions used mathematics problems modeled after three levels of complexity in problem content (Bruner, 1968), representation, representation and solution in a graphic, using a different graphic for each category with small numbers, and symbolic problem solving, generalizing the process to problems with large numbers. In this phase, students were instructed to substitute large numbers for small numbers to achieve a better understanding of the problem and to choose the mathematical operation that enabled them to solve it correctly. The 49 subjects in the Control Group were not given the training program. They remained in the classroom solving the arithmetic problems from the standard third-grade curriculum.

Analyses

Two types of statistical comparisons were made, a comparison of the differences in problem-solving performance obtained between the reflective and the impulsive students using the Mann-Whitney U test, and an analysis of variance.

Results

The first statistical comparison was performed on the subjects classified as having Impulsive and Reflective styles, independent of their group affiliation. Differences were found between the two groups of students for the problems with small numbers. Differences were statistically significant ($p < .05$) between the Reflective and Impulsive groups in both Experimental and Control groups for problems with small numbers ($U = 7.65$; $p < .05$), for problems with large numbers ($U = 55$; $p < .01$), and for the total number of problems solved ($U=63$; $p<.01$). No significant differences were found within the Experimental Group between the Reflective and Impulsive subjects, or within the Control Group for problems with small numbers ($U=54.5$; ns), for problems with large numbers ($U=54.5$; ns), or for the total number of problems solved ($U=56$; ns).

TABLE 2
MEANS AND STANDARD DEVIATIONS FOR NUMBERS OF PROBLEMS SOLVED BY REFLECTIVE AND IMPULSIVE STUDENTS FOR PROBLEMS USING LARGE AND SMALL NUMBERS AND ALL PROBLEMS FOR EXPERIMENTAL AND CONTROL GROUPS

	Numbers of Problems Solved					
	Reflective Group			Impulsive Group		
	Using Small Numbers	Using Large Numbers	All Problems	Using Small Numbers	Using Large Numbers	All Problems
Control Group						
M	17.7	13.3	39.6	16.1	12.1	37.2
SD	2.2	1.9	2.6	1.7	2.0	1.8
Experimental Group						
M	15.6	11.3	33.2	14.2	10.3	31.3
SD	1.0	2.1	1.3	2.2	1.0	2.1

The variances between the Control and Experimental groups were analyzed using analyses of variance; see Table 3. The Reflective students of the Experimental Group did not differ significantly from Reflective students in

TABLE 3
ANALYSIS OF VARIANCE FOR EXPERIMENTAL VS CONTROL GROUPS AND REFLECTIVE VS IMPULSIVE STUDENTS

Source	*df*	*SS*	*MS*	*F*
Reflective Students: Experimental vs Control				
Between Subjects	13	1372.8	105.6	.77
Within Subjects	14	1912.0	136.5	
Treatments	1	289.2	289.2	2.31
Residual	13	1622.7	124.8	
Total	27	3284.8		
	M_{diff}	Fisher	Scheffé *F*	Dunnett *t*
	− 6.42	9.12	2.31	1.52
Impulsive Students: Experimental vs Control				
Between Subjects	11	976.3	88 7	.60
Within Subjects	12	1757.0	146 4	
Treatments	1	748.1	748.1	8.15*
Residual	11	1008.8	91.7	
Total	23	2733.3		
	M_{diff}	Fisher	Scheffé *F*	Dunnett *t*
	−11.61	8.60	8.01	2.85

*$p<.05$.

the Control Group on the problem-solving measures (*M* correct responses for the Experimental Group = 33.2; *M* for the Control group = 39.6; ns). On the other hand, the mean differences in number of correct responses for the Impulsive students of the Experimental Group were statistically significant when compared to the students from the Control Group. In this situation, the Impulsive students of the Experimental Group had a lower mean score on the Matching Familiar Figures Test than the Impulsive students in the Control Group ($F = 8.15$, $p < .01$).

Discussion

It appears that problem solving may be negatively associated with the Impulsive cognitive style as suggested by the Control Group's data. Impulsive students improve in arithmetic problem solving significantly when given the training program. The training program in problem-solving strategies did not improve the performance of arithmetic problem solving in the Reflective students. It is quite possible to infer that such an effect occurs because these students do not need the training. It may be that the Reflective students already know how to decode successfully a simple arithmetic problem, and they do not need to learn a new problem-solving strategy to solve problems correctly (see Table 3: Reflective students in the Experimental vs Control groups. Fisher = 9.12, ns; Scheffé = 2.31, ns; Reflective vs Impulsive students in the Control Group: Fisher = 8.6, $p < .05$; Scheffé = 8.01, $p < .05$).

Students with an Impulsive cognitive style benefited the most from the training in problem solving used in this study. The source of this could be any of several effects. The most likely and simplest explanation is that the experimental training program helps impulsive students change their problem-solving strategies so that they can correctly solve the arithmetic problems. The same effects can be seen when the comparison is performed between experts and beginners in arithmetic problem solving. The Impulsive expert problem solvers were not differentiated by the total number of problems solved when compared to the Reflective students regardless of whether they were beginners or experts (Aguilar, 1995). As one might expect, it does not appear that the students' cognitive styles change significantly as the result of this type of training program in problem solving. Conversely, the Reflective students did not significantly improve their problem-solving performance after training. Reflective students may already have known and used the kinds of strategies taught while Impulsive students did not.

Reflective students tend to use an analytical thinking strategy, defined by focusing attention and differential validation of many response opportunities. There is no agreement about the strategy used by more impulsive students. One hypothesis is that they use a holistic strategy (Tinajero & Paramo, 1993), and most activities taught in school settings (reading, writing, solving problems, etc.) require strategies for analytical and sequential information processing (Cohen, 1969).

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APPENDIX

Examples of some Arithmetic Word Problems:

Change: Amy had two candies. Mary gave her three more candies. How many candies does Amy have now?

Combine: Amy has two candies. Mary has two candies. How many fewer candies do they have altogether?

Compare: Mary has three candies. Amy has two candies. How many fewer candies does Amy have than Mary?

Equalize: Mary has five candies. Amy has two candies. How many candies does Amy have to buy to have as many candies as Mary?

Isomorphism: On a dance floor there are five groups with six dancers each. How many dancers are on the floor?

Large Escalates: Mr. Brown fills the tank of his car with 17 liters of gas. Mr. Miller fills his with three times as much. How much gas does Mr. Miller take?

Small Escalates: Mr. Brown fills the tank of his car with 51 liters of gas. Mr. Miller fills his with three times less. How much gas does Mr. Miller take?

Cartesian Product: In a transport company there are 13 trucks and 18 trailers. How many combinations of trucks and trailers can be put together?